

SETON HALL UNIVERSITY
TWENTYTHIRD ANNUAL
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MATHEMATICS COMPETITION

1. One number is three more than a second number. Three times the first number is one less than four times the second number. Find the two numbers.

ANS: 13, 10

2. Find the number of integers x for which $-5 < \sqrt[3]{x+7} < 10$.

ANS: 1124

3. How many different arrangements can be made of the letters in the word abscissa?

ANS: 3360

4. How many quarts of water must be added to 30 quarts of a 25% acid solution in order to obtain a 15% acid solution?

ANS: 20

5. An airplane made a trip of 815 miles in 4 hours. Part of the trip was made at 180 miles per hour and the remainder at 220 miles per hour. Find the number of miles traveled at the slower rate.

ANS: 292.5

6. A three digit positive integer is 40 more than 14 times the sum of its digits three digits. If the number is divided by the sum of its three digits, the quotient is 16 and the remainder is 6. If the number is divided by twice the sum of its first and third digits, the quotient is 13 and the remainder is 18. Find the number.

ANS: 278

7. Two perpendicular planes P_1 and P_2 , in 3-dimensional space, intersect in line ℓ . Plane P_3 passes through point A on line ℓ and is perpendicular to line ℓ . Points in space which lie on two of the planes P_1 , P_2 and P_3 and are less than one inch from point A or are between 3 inches and 5 inches from point A lie on line segments. Find the sum of the lengths of these line segments.

ANS: 18

8. The real numbers x for which $\frac{4x^4 + 4x^3 + 21x^2 + 45x - 45}{4x^2 + 9x - 9} \leq 5$ lie on an interval on a real number line. Find the length of the interval.

ANS: $3\frac{3}{4} = 15/4$

9. Find all real values of x such that x is an integer multiple of $1/3$ and $(6x+31)^x = 1/9$.

ANS: $-2/3$

10. Find all real values of x for which $27 \cdot 3^{4x} - 12 \cdot 3^{3x} - 728 \cdot 3^{2x} + 324 \cdot 3^x - 27 = 0$.

ANS: -1, -2, $3/2$

11. Six (distinguishable) dice are thrown. What is the probability of getting a sum greater than 32? Give the answer as a reduced fraction.

ANS $7/3888$

12. An angle with vertex A has degree measure 120° . Point C lies on the initial side of angle A and is 6 inches from A; point B lies on the terminal side of angle A and is 4 inches from point A. Find the value of $\sin ABC$.

ANS: $3\sqrt{57}/38$

13. Find all pairs (x,y) of real numbers for which $3x^2 - 2xy = 4$ and $x^2 - 2xy + y^2 - x + y = 0$.

ANS: $(2,2), (-1+\sqrt{5}, -2+\sqrt{5}), (-2,-2), (-1-\sqrt{5}, -2-\sqrt{5})$

14. Let n be a positive integer and define $F(n)$ to be the number of ordered pairs (x,y) such that x and y are both positive integers and $xy \leq n$. (For example $F(4) = 8$ since for the ordered pairs $(x,y) = (1,4), (1,3), (1,2), (1,1), (2,2), (2,1), (3,1),$ and $(4,1)$, $xy = 4, 3, 2, 1, 4, 2, 3,$ and 4 , respectively.) Find the value of $F(81)$.

ANS: 373

15. The sum of the first four terms in a geometric progression is 255. The product of the second term and the reciprocal of the fourth term is 4 less than the third term. Given that all terms in the geometric progression are rational numbers, find the sum of the first and fourth terms in the progression.

ANS: 315

16. Three vertices of a rhombus ABCD lie on a circle of radius 6 inches, and the fourth vertex is outside the circle. The perimeter of the rhombus is three times the length of a diameter of the circle. Find the area of rhombus ABCD.

Ans: $243\sqrt{7}/8$